Math in Idaho - Mining: Inverse Distance Weighting Estimation

Background Information

This lesson is designed to engage students in high school level mathematics as applied in the mining industry. This lesson is best situated in the Geometry/Alg 2 or Int 2/Int 3. Students will need to have a basic understanding of summation notation and rational expressions.

Lesson Duration

Teachers should use their discretion to keep pace with the needs of students. This introductory lesson is designed for 50 minutes, but this lesson could easily be extended for a longer exploration.

Instructional Sequence Summary

This lesson is built to engage students in guided-inquiry using the 5E lesson model. In this model, there are 5 phases that students will experience; (1) **Engage**: The teacher provides context to engage the students in curiosity and surface existing knowledge through class discussion, (2) **Explore**: The students collaborate to investigate targeted questions from the context; (3) **Explain**: The teacher leads a whole class discussion making explicit connections between student's thinking from the explore phase and formalizing the mathematics; (4) **Elaborate**: The teacher provides context or discussion to generalize the important concepts and their connections beyond the original lesson context; (5) **Evaluate**: Teacher provides students with a brief assessment to gauge understanding and inform future instruction.

Standards alignment:

A.SSE.A. Interpret the structure of linear, quadratic, exponential, polynomial, and rational expressions.

F.IF.B.4. Interpret functions that arise in applications in terms of the context, including rational functions.

Student Learning Targets

I can calculate the estimated mineral concentration using the "Inverse Distance Weighting" formula.

I can explain the use of multiplicative inverses "weight" the formula.

Assessment:

Students will be given an exit ticket, being asked to apply the formula and explain some of the reasoning behind its complexity.

Key Vocabulary:

Mining: Mineral Concentration, Inverse Distance Weighting Method
Math: Inverse, (Summation Notation)

Important Questions:

How can we predict the mineral concentration to decide where to mine?

Lesson Plan/Learning Experience

Engage (5-10 minutes)

Students are shown the video about Mining in Idaho. The instructor then leads a discussion about the decision of where to dig for resources, pressing students to connect why proper planning is important to make sure that there are enough materials to justify the dig (if students don't bring up the ideas, the instructor can point to the cost of digging and commitment to choosing a site to dig and mine). Finally, an open discussion for students to predict how this estimation of resources might occur.

This lesson can be taught to build upon, Mining in Idaho Lesson #1, but it is not necessary to teach that first. Whether as a review from that lesson or as a quick background build, the instructor should bring up the "Nearest Neighbor Method" (NNM) as an example of an oversimplistic way for predicting mineral concentration. In the NNM, you simply find the distance to existing sample points and the concentration prediction is whatever the concentration of the nearest sampling point. This launch should conclude with the desire to have a more accurate estimation.

Explore (25-30 minutes)

A mining company maps out its mine using a Cartesian coordinate plane with each axis scaled by one meter. The following table shows the four locations that have been sampled and the Cooper concentration that was found at that location:

Sample Site	Location on Grid (x,y)	Cooper Concentration (%)
A	(2,1)	4.0%
B	(5,2)	6.0%
C	(3,4)	5.0%

They have targeted a location at (4,3) for the next dig. Use all three samples to come up with a solid prediction of the new location's concentration.

As students work the teacher will press them on these ideas:

> How can you differentiate between the three sites in your calculation?
>
> Are sample sites that are closer to the desired location treated differently than those that are farther away?

Explain (Together With Explore Section)

The instructor guides a consolidation discussion to connect ideas that arose around the classroom and find the similarity in student thinking. Specifically, the teacher should highlight how any ideas of averaging came up and anyway to factor in the distance. With these two aims established, the instructor formally introduces the "Inverse Distance Weighting Formula" and the class collaboratively calculates the IDW for the given problem.

$$C = \frac{\sum \left(\frac{C_i}{d_i^2}\right)}{\sum \left(\frac{1}{d_i^2}\right)}$$

Where C_i is the concentration of a given point *i* and d_i is the distance between point *i* and the desired point.

Elaborate (10-15 minutes)

The idea behind the elaboration is to expand the connection beyond this specific topic to the broader math landscape. This can also include expanding beyond this specific context to larger contexts. The goal is to deepen student understanding through further connection and engagement. The following questions are offered for consideration, but do not all need to be covered.

- How does dividing by the distance provide differing "weights" to the various locations? (Scaffolding: does a larger distance make the resulting quotient larger or smaller?)
- Why is each distance used twice in the calculation, in both the numerator and denominator? (Here we can press the idea of inverse, how we need to "undo" the division by distance to get back to a reasonable estimation of concentration.)
- How does the estimation change if you don't square the distance? Why do you think squaring the distance can create a more accurate estimation?

Evaluate (5 minutes)

Students will be given an exit ticket. There is procedural option and a conceptual option below:

Procedural:

A mining company is evaluating a region for its **nickel** content. They have taken samples from two locations and want to estimate the concentration of nickel at a new location using the **Inverse Distance Weighting (IDW)**.

Here are the sample points:

Sample Location (x, y)	**Nickel Concentration (%)**

(1, 3)	1.2%
(4, 1)	2.0%

The company wants to estimate the nickel concentration at **location (3, 4)**.

Conceptual:

Describe how the formula for Inverse Distance Weighting creates “weights” so that closer points have more influence over the estimation.